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Vibration analysis and response characteristics of a half car model subjected to different sinusoidal road excitation

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Abstract

The displacement response of different masses of half car model. The analysis has been done for different car models also to see the dynamic response of the driver body coupled with the seat of a vehicle. It has been assumed the driver body is rigidly coupled with seat of the vehicle. The vehicle has been modeled for two D.O.F, in two D.O.F Half car model two motion (Pitch and Bounce) have been considered. The response of the vehicle has been obtained for different velocities and different amplitudes sinusoidal bump excitation.

Key words: Half car Model, Degree of Freedom, Model development, Lagrange equation

1. Introduction

This paper deals with the dynamic characterization of an automotive shock absorber, a continuation of an earlier work Vibration is undesirable, not only because of the unpleasant motion, the noise and the dynamic stresses, which may lead to fatigue and failure of the structure, but also because of the energy losses and the reduction in performance which accompany the vibrations [1-2]. Vibration analysis should be carried out as an inherent part of the design because of the devastating effects, which unwanted vibrations could have on machines and structures. The shock absorber is one of the most important elements in a vehicle suspension system. It is also one the most non-linear and complex elements to model. The current method of characterizing the dynamic properties of shock absorbers for CAE models involves testing at discrete frequencies, displacements, and preloads using an MTS test machine. The dynamic stiffness (K) and damping (C) are extracted by fitting a linear model of the form F(w)=K*x(w)+C*v(w) to the measured input displacement (x), velocity (v), and output force (F). The excitation technique is a pure sine excitation at the desired frequency and amplitude. These harmonic excitations are then swept through all desired frequency and amplitudes. First, it is commonly understood and accepted that human response to dynamic excitation depends on many mechanical, physical, physiological and psychological parameters [3]. The biodynamic response characteristics of seated occupants influenced by several factors, among which body posture, body weight and vibration excitation type and amplitude probably represent the most influential parameters [4]. Half car has been modelled as two DOF systems, in which bounce and pitch motion has been considered, the driver body and vehicle body has considered as one mass. The mathematical analysis of the suspension system has been performed to develop the model. Dynamic analysis has been performed, for solving the half car model. The goal of this study was to determine if the current excitation technique holds true when more than one frequency is present. In recent years, commercial demand for comfortable and quiet vehicles has encouraged the industrial development of methods to accommodate a balance of performance, efficiency, and comfort levels in new automobiles. Particularly, the noise, vibration, and harshness (NVH) characteristics of cars and trucks are becoming increasingly important [5].

2. Mathematical Modeling

2.1 Half Car Model: Two Degree of Freedom

The vehicle mass is set at 750kg, and the mass of human body is assumed to be 55.2kg coupled with the vehicle body. It has a moment of inertia about the center of mass of

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805.2kg·m^2. Figure A: The center of mass location is set at a distance 1m from the front axle and 1.5m from the rear axle. The effects of the tires (rolling motion, mass, etc.) have been neglected [6]. The equivalent stiffness and damping of the front and rear axle assemblies are set to the same values for each axle, k_1=k_2=25,000 N/m, and c_1=c_2 =2,000N·s/m respectively. The road roughness has not been considered in this analysis

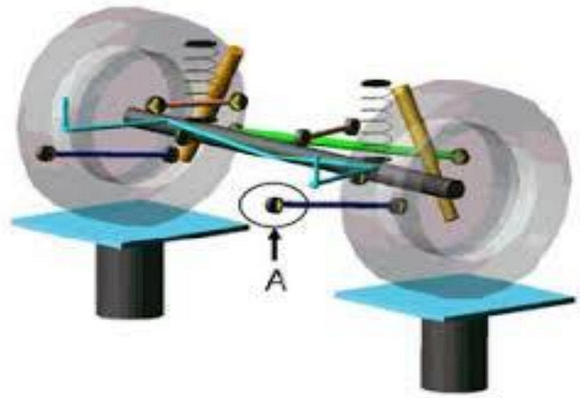


Figure A: Half car model with 5-link rear suspension

Table 1 Modes of Vibration for Various Car Models

Modes	Bounce	Pitch	Roll	Yaw
Half Car Model	Yes	yes	no	No

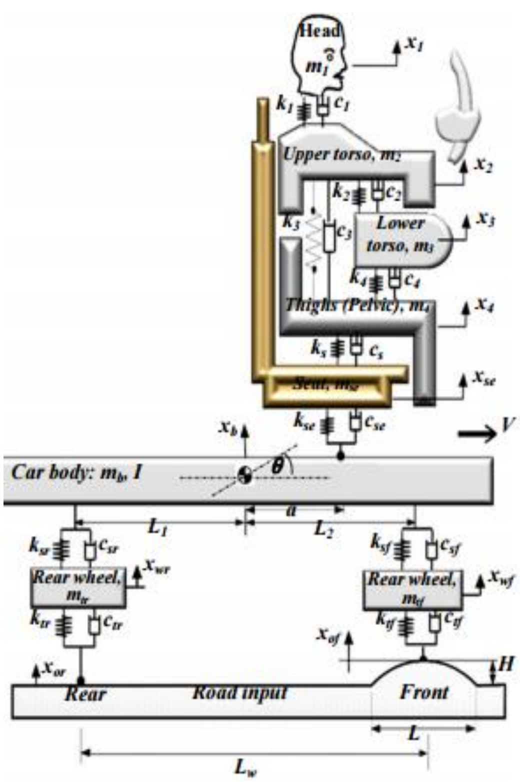


Figure 1: Two-degree-of-freedom half car mode

Table 2: mass-spring-damper two D.O.F model

Variable	Description	Value
M_1	Mass of the automobile	750kg
M_2	Mass of the driver body	55.2kg
M	Total mass(M_1+M_2)	805.2kg
r_g	Radius of gyration	1m
J	The mass moment of inertia of the automobile about the center of mass	805.2kg-m^2
c_1	Damping coefficient of the dashpot on the front of the model	2000Ns/m
c_2	Damping coefficient for the dashpot on the rear of the model	2000Ns/m
k_1	Spring coefficient for the spring at the front of the model	25000N/m
k_2	Spring coefficient for the spring at the rear of the model	25000N/m
l_1	Distance from the center of mass to the front spring/damper	1m
l_2	Distance from the center of mass to the rear spring/damper	1.5m

3. Governing Equations

To determine the equations of motion, Lagrange's equations, also known as the energy method, has utilized. Equation (1) shows the general form of Lagrange's equations

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}i}\right) - \frac{\partial L}{\partial qi} = Qi, \qquad \text{i} = 1,2 \tag{1}$$

Where $Q_i$is the non conservative generalized force corresponding to the ith generalized coordinate .Where L is related to kinetic and potential energies as follows in equation (2)

$$L = T - U \tag{2}$$

Where T is the kinetic energy, U is the potential energy of the system. The terms q_i and $Q_i$from Eq. (1) represents a degree of freedom and the non-conservative work for each DOF (subscript $i$denoting the first and second degrees of freedom); represents the derivative ofq_i.

By substituting equation (2) into equation (1) for the value of L leads to equation (3).

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} = Q_i \tag{3}$$

These equations are then used with the two degree of freedom system of the automobile. First, the generalized coordinates replace the representationq_i. These coordinates are as follows.

q_1=x(t)

q_2=θ(t)

The kinetic energy and potential energy equations developed as follows in equations (4) and (5), as well as the force acting on the vehicle.

$$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}J\dot{\theta}^2 \tag{4}$$

$$U = \frac{1}{2}k_1(x - l_1\theta - y_1)^2 + \frac{1}{2}k_2(x + l_2\theta - y_2)^2 \tag{5}$$

$$F = \frac{1}{2}c_1(\dot{x} - l_1\dot{\theta} - \dot{y}_1)^2 + \frac{1}{2}c_2\left(\dot{x} + l_2\dot{\theta} - \dot{y}_2\right)^2 \tag{6}$$

where k_1 and $k_2$are the equivalent spring rates of the front and rear suspension, x is the displacement of the body's center of gravity, l_1 and $l_2$are the distances from the center of gravity to the front suspension and rear suspensions, and $y_1$and y_2 are the input functions of the road for the front and rear of the system.

The Lagrange equation for x found through the following series of steps.

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}} = m\dot{x}\right) = m\ddot{x}$$

$$\frac{\partial T}{\partial x} = 0$$

$$\frac{\partial U}{\partial x} = k_1((x - l_1\theta - y_1)) + k_2(x + l_2\theta - y_2)$$

$$Q_1 = -\frac{\partial F}{\partial \dot{x}} = -(c_1\left(\dot{x} - l_1\dot{\theta} - \dot{y}_1\right) + c_2\left(\dot{x} + l_2\dot{\theta} - \dot{y}_2\right))$$

These separate parts placed into the Lagrange equation as follows.

$$m\ddot{x} + k_1(x - l_1\theta - \mathrm{y1}) + k_2(x + l_2\theta - y_2) = -(c_1\left(\dot{x} - l_1\dot{\theta} - \dot{y}_1\right) + c_2\left(\dot{x} + l_2\dot{\theta} - \dot{y}_2\right))$$

This equation has been expanded in order to place it in matrix form, separating the motion of the car in $x$andθ coordinates, from the variables of motion from the road, $y_1$and y_2. The expanded form follows in equation (7).

$$\begin{aligned} m\ddot{x} + (c_1 + c_2)\dot{x} + (k_1 + k_2)x + (-c_1l_1 + c_2l_2)\dot{\theta} \\ + (-k_1l_1 + k_2l_2)\theta \\ = \dot{y}_1c_1 + \dot{y}_2c_2 + y_1k_1 + y_2k_2 \end{aligned} \tag{7}$$

The same series of steps are performed on equations (4)-(6) for $q_2$and θ.

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}} = J\dot{\theta}\right) = J\ddot{\theta}$$

$$\frac{\partial T}{\partial \theta} = 0$$

$$\frac{\partial U}{\partial \theta} = k_1(-l_1)(x - l_1\theta - y_1) + k_2(-l_2)(x + l_2\theta - y_2)$$

$$\begin{aligned} Q_2 = -\frac{\partial F}{\partial \dot{\theta}} = -(c_1\left(\dot{x} - l_1\dot{\theta} - \dot{y}_1\right)(-l_1) \\ + c_2\left(\dot{x} + l_2\dot{\theta} - \dot{y}_2\right)(-l_2)) \end{aligned}$$

these equations are placed in the Lagrange form.

$$\begin{aligned} J\ddot{\theta} + k_1((-l_1)(x - l_1\theta - y_1)) + k_2(-l_2)(x + l_2\theta - y_2) \\ = -(c_1\left(\dot{x} - l_1\dot{\theta} - \dot{y}_1\right)(-l_1) \\ + c_2\left(\dot{x} + l_2\dot{\theta} - \dot{y}_2\right)(-l_2)) \end{aligned}$$

These equations are expanded to be in the same form as equation (7).

$$\begin{aligned} J\ddot{\theta} + (c_1l_1^2 + c_2l_2^2)\dot{\theta} + (k_1l_1^2 + k_2l_2^2)\theta + (-c_1l_1 + c_2l_2)\dot{x} \\ + (-k_1l_1 + k_2l_2)x \\ = \dot{y}_1(-c_1l_1) + y_1(-k_1l_1) + \dot{y}_2(c_2l_2) \\ + y_2(k_2l_2 \end{aligned} \tag{8}$$

The equation of motion is then set up in matrix form by combining equations (7) and (8) to make equation (9).

$$\begin{aligned} \begin{bmatrix} m & 0 \\ 0 & J \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_1l_1 + c_2l_2 \\ -c_1l_1 + c_2l_2 & 975c_1l_1^2 + c_2l_2^2 0 \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{\theta} \end{bmatrix} \\ + \begin{bmatrix} k_1 + k_2 & -k_1l1 + k_2l2 \\ -k_1l_1 + k_2l_2 & k_1l_1^2 + k_2l_2^2 \end{bmatrix}\begin{bmatrix} x \\ \theta \end{bmatrix} \\ = \begin{bmatrix} c_1 & c_2 \\ -c_1l_1 & c_2l_2 \end{bmatrix}\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} \\ + \begin{bmatrix} k_1 & k_2 \\ -k_1l_1 & k_2l_2 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \end{aligned} \tag{9}$$

$$J = m * rg2 \tag{10}$$

The parameters of the system are as follows: k_1 = k_2 =25000N/m, c_1 = c_2 = 2000 Ns/m, M =805.2kg, J = 805.2kgm^2,r_g = 1,l_1 = 1m, and l_2 = 1.5m. Substituting these values and expanding Eqs. (3.9) & (3.10) yields Eqs. (11) & (12)

$$\begin{aligned} \ddot{x} + 4.97\dot{x} + 62x + 1.24\dot{\theta} + 15.5\theta \\ = 2.48\dot{y}_1 + 31y_1 + 2.48\dot{y}_2 \\ + 31y_2 \end{aligned} \tag{11}$$

$$\ddot{\theta} + 8.07\dot{\theta} + 101\theta + 1.24\dot{x} + 15.5x = -2.48\dot{y}_1 - 31y_1 + 3.73\dot{y}_2 + 46.57y_2 \quad (12)$$

The car is assumed to be travel over road at different speeds, the road is assumed to be sinusoidal in nature with amplitude of X (in meters) and having a wavelength (d) of 5meters. With this information, the input functions $y_1$and y_2 are defined in Equation. (13) & (14)

$$y_1 = X\sin(\omega t) \quad (13)$$

$$y_1 = X\sin(\omega t + \phi) \quad (14)$$

Where, ϕ is the phase difference between front and rear wheel.

$$\phi = 2\pi\,(l_1 + l_2)/d$$

$$\phi = 2\pi\ (1 + 1.5)/5 = \pi$$

Where, *t* is the time traveled and πis the time shift that accounts for the time that it takes for the rear suspension to negotiate the "bump" that the front suspension had negotiated.

3.1 Calculation for the Natural Frequencies

$$\ddot{x} + 62x + 15.5\Theta = 0 \quad (15)$$

$$\ddot{\theta} + 101\theta + 15.5x = 0 \quad (16)$$

$$x = X\sin(\omega t + \phi) \quad (17)$$

$$\theta = \Theta\sin(\omega t + \phi) \quad (18)$$

On putting the value of equation (17) & (18) in equation (15) & (16) yield equation (19).

$$\omega^4 - 163\omega^2 + 6021.75 = 0 \quad (19)$$

On solving equation (19) the value of natural frequencies are as follow:

$\omega_1 = 7.5$

$\omega_2 = 10.3$

Corresponding to these natural frequencies, the value of amplitude ratios are as follow.

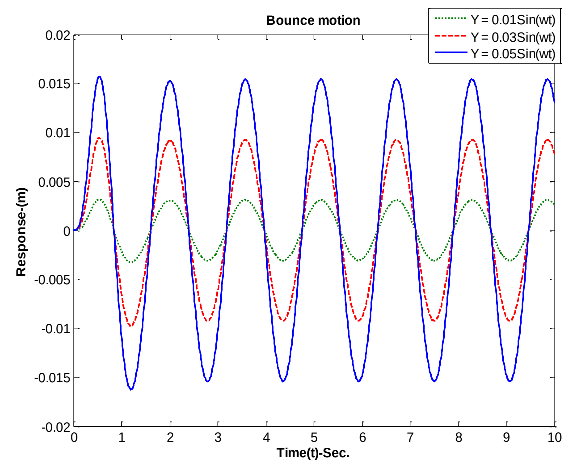
$$\frac{X_1}{\Theta_1} = -2.86$$

$$\frac{X_2}{\Theta_2} = 0.3$$

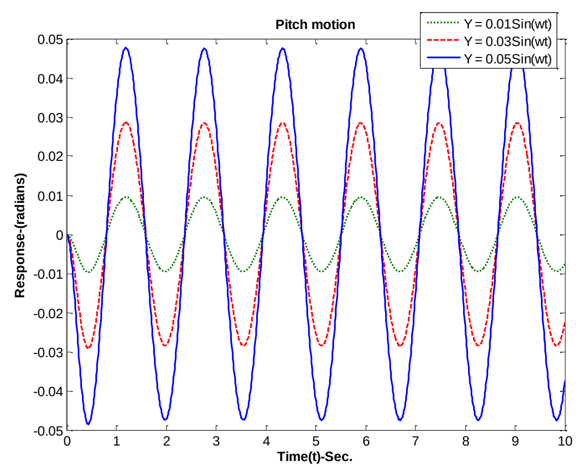
4. Results and Discussion

To obtain the dynamic response of the system MATLAB program has been written for equations (11) and (12), for different velocities 11.5km/hr, 17.2km/hr, 22.9km/hr and 28.65km/hr respectively.

First of all, the half car model encountering to sinusoidal bump analysis has been done to see the response. When the amplitude of sinusoidal bump is increased the amplitudes of vibration are also increased. The amplitudes of vibration magnifies than the input sinusoidal excitation from road to the wheel. On increasing the velocity of the vehicle the amplitudes of vibration are increased up to the certain velocity and for a particular velocity of the vehicle the amplitudes of vibration reaches to a maximum value. If the velocity of the vehicle is further increased then the vibration amplitudes are continuously decreased.

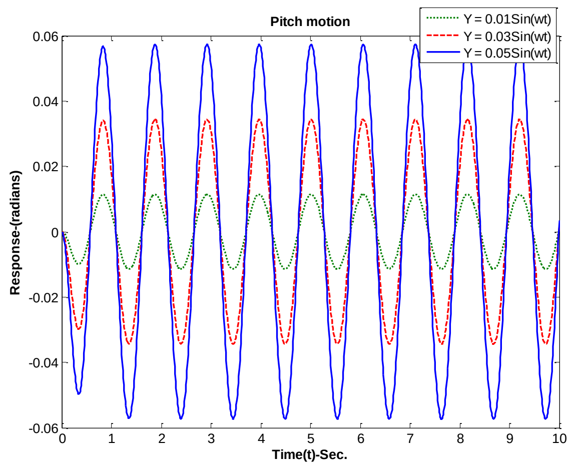


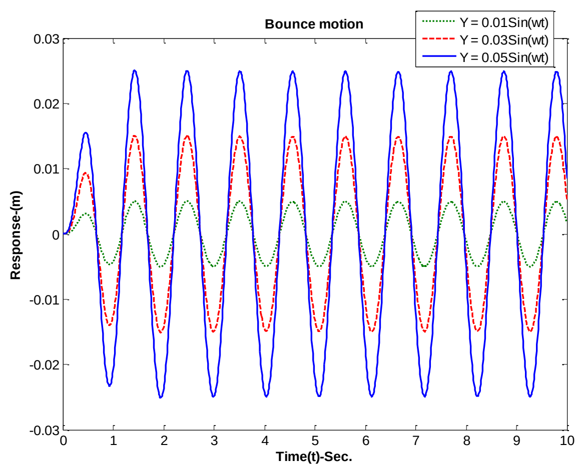
(a)



(b)

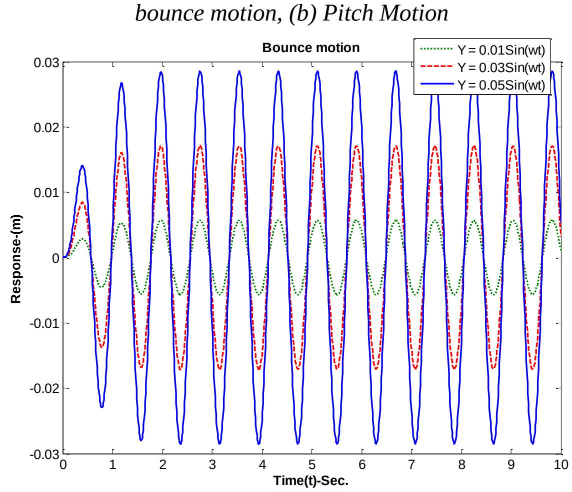
Figure 1: Motion response of the vehicle at v=11.5km/h, (a) bounce motion, (b) Pitch Motion





(b)

Figure 2: Motion response of the vehicle at v=17.5km/h, (a) bounce motion, (b) Pitch Motion



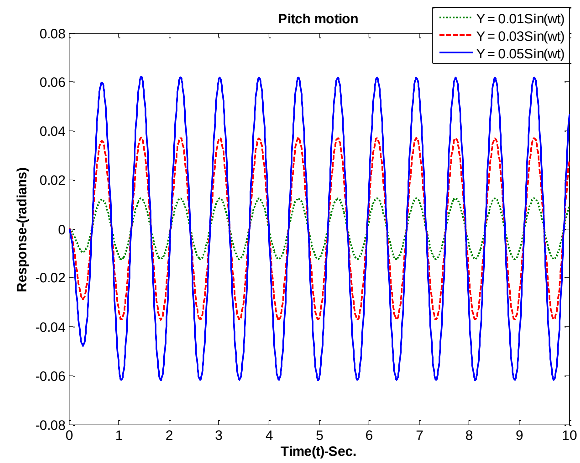
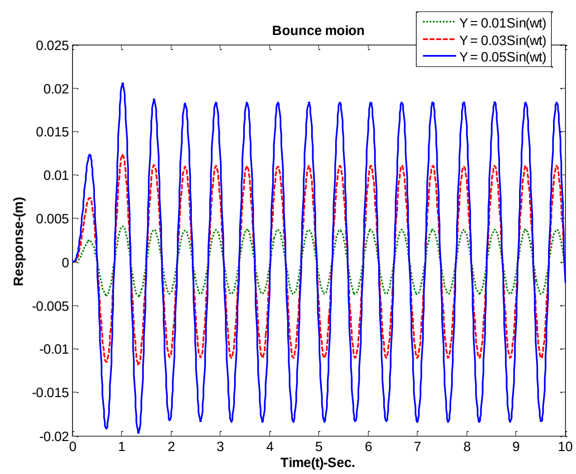
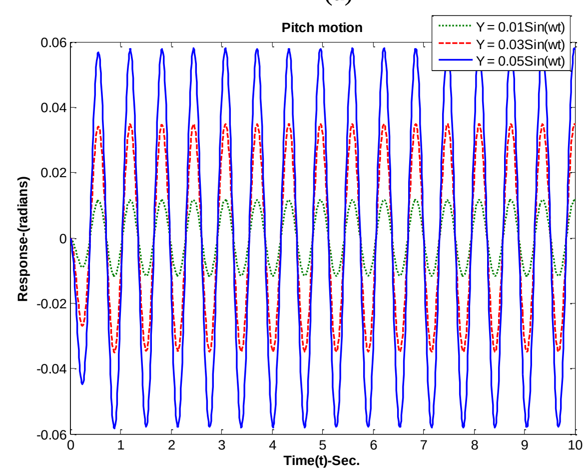


Figure 3: Motion response of the vehicle at v=22.9 km/h, (a) Bounce motion, (b) Pitch Motion



(a)



(b)

Figure 4: Motion response of the vehicle at v=28.65 km/h, (a) Bounce motion, (b) Pitch Motion

5. Conclusion

In this paper, a stochastic half-car model is used to investigate the dynamic response of half cars with uncertainty. The effect of uncertainty in the vehicle's parameters on the randomness of the natural frequencies and vehicle's random responses are presented by using the MATLAB CODES. The dynamic characteristics and random response of stochastic vehicles are obtained expediently. This method will also be applied to the dynamic analysis of random vehicles by using stochastic full-car models

The amplitudes of vibration of a vehicle can be controlled for a particular speed with particular values of damping and stiffness coefficients. If the values of damping and stiffness coefficients increase to high extent then spring and damper act as rigid body and all input vibration to the tire transmitted to the upper parts of the vehicle as such. The effect of spring coefficients is not so much on vibration control to the different parts but variation in damping coefficient is having much effect on vibration amplitudes. From the results it has seen that the transmission of vibration to the upper part is of maximum amplitudes. The amplitude of vibration is minimum to the tire. It has been observed from the results that the bump amplitude to the road should be of optimum value.

Random vibration analysis can be further done of a road vehicle is investigated using different car models which are quarter car model, bicycle car model, and half car model. Computer programs in Mathematical are developed for all car models. To understand the base excitation response behaviors of the sprung mass in all car models, firstly deterministic vibration analysis are carried out and the results are presented by graphs. This graphs show the vibration amplitudes vs time under the excitation frequencies which are near to and far from the natural frequencies. To simplify the calculations, proportional damping is considered for damping properties in car models.

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