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Properties of gsp-Hausdorff spaces in topology

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Abstract

In this paper, we define and study gsp- Hausdorff spaces and allied Hausdorff spaces, namely, gp-Hausdorff spaces, αg-Hausdorff spaces, rps–Hausdorff spaces and semipre-hausdorff spaces. Also, we define and study their comparative and preserving properties

Keywords: Semi preopen sets, gsp-closed sets, preopen sets, gs-closed sets,rps-closed sets, gsp-irresoluteness, pre-gsp-continuous functions and rps-irresolute functions.

1. Introduction

In 1995,J. Dontchev [4] has defined and studied the concepts of gsp-closed sets , gsp-continuity and gsp-irresoluteness in topological spaces.In 1993 and 1998 , resp. , H.Maki et al [7] and R.Devi et al [3] have defined studied the concepts of αg-closed sets and αg-irresolute functions in topology. In 1998 , 1999 and 2002 ,resp., T.Noiri et al [14] , Arokiarani et al.[2] and Park et al [16] have defined and studied the concepts of gp-closed sets , gp-continuity, gp-irresoluteness and pre-gp-continuity in topology. In 2009, Navalagi et al [11] have defined and studied the concept of strongly semipre-continuous functions in topology. In 2010, 2011, resp., T. Shyla Isac Mary et al [18& 19] have defined and studied the concepts of rps-closed sets, rps –irresolute functions in topology. In 2014, Navalagi et al [12] have defined and studied notion of pre-gsp-continuous functions. In this paper, we define and study gsp- Hausdorff spaces and allied Hausdorff spaces like gp-Hausdorff spaces, αg-Hausdorff spaces, rps –Hausdorff spaces and semipre-Hausdorff *spaces, a*lso, we define and study their basic properties.

2. Preliminaries

Throughout this paper (X, τ) and (Y, σ) (or simply X and Y) denote topological spaces on which no separation axioms are assumed unless explicitly stated . If A be a subset of X, the closure of A and the interior of A is denoted by Cl (A) and Int (A), respectively. A subset A of a space X is called regular open (in brief, r-open) if A =Int Cl (A) and regular closed (in brief, r-closed) if A = Cl Int (A).
We give the following define are useful in the sequel.

Definition 2.1: The subset of A of X is said to be

(i) A pre-open (in brief, p-open) [8], set, if A⊂ Int (C*l*(A))
(ii) A semi-pre-open [1] set , if A⊂ Int (C*l*(A))
(iii) α-open [13] set, if A A⊂ Int (C*l*(A))

The compliment of a p-open (resp., semipreopen , α-open) set is called p-closed [5] (resp., semipreclosed [1], α-closed [9]) set in space X . The family of all pre-open (resp. semipre-open, α-open) sets of a space X is denoted by PO(X) (resp., SPO(X) ,αO(X)) and that of pre-closed (resp.semipre-closed ,α-closed) sets of a space X is denoted by PF(X),
(resp.SPF(X) α F(X)).

Definition 2.2[5]: The intersection of all pre-closed sets of X containing subset A is called the pre-closure of A and is denoted by pCl (A).

Definition 2.3[1]: The intersection of all semipre-closed sets of X containing subset A is called the semipre-closure of A and is denoted by spCl(A).

Definition 2.4[9]: The intersection of all α-closed sets of X

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containing subset A is called the α-closure of A and is denoted by αCl(A).

Definition 2.5[5]: The union of all pre-open sets of X contained in A is called the pre-interior of A and is denoted by pInt (A).

Definition 2.6[1]: The union of all semipre-open sets of X contained in A is called the semipre-interior of A and is denoted by spInt(A).

Definition 2.7[9]: The union of all α-open sets of X contained in A is called the α-interior of A and is denoted by αInt (A).

Definition 2.8: A sub set A of a space X is said to be

(i) A generalized closed (briefly, g- closed) [6] set if Cl(A)⊆U ,whenever A)⊆U and U is open set in X .
(ii) A α- generalized closed (briefly, αg- closed) [7] set if αCl(A))⊆U whenever A)⊆U and U is open set in X.
(iii) A regular generalized closed (briefly, rg-closed) [15] if Cl(A) ⊂ U , whenever A ⊂U and U is r-open in X .
(iv) A generalized semi-preclosed (briefly, gsp- closed) [4] set if spCl(A))⊆U whenever A)⊆U and U is open in X .
(v) A generalized pre -closed (briefly, gp- closed) [14] set if pCl(A))⊆U whenever A⊆U and U is open in X .
(vi) a regular presemiclosed (briefly ,rps-closed) set [18] if spCl(A) ⊂ U , whenever A ⊂ U and U is rg-open in X.

The complement of a g-closed (resp, αg-closed, rg-closed , gsp-closed, gp-closed ,rps-closed) set in X is called g-open (resp. αg-open, rg-open ,gsp- open, gp- open , rps-open) set in X. The family of all gsp-open sets of X is denoted by GSPO(X).

Definition 2.9: A function f : X→ Y is called

(i) Semi pre-continuous [10] if the inverse image of each open set of Y is semipreope in X.
(ii) Strongly semi pre-continuous [11] if the inverse image of each semi-preopen set of Y is open in X.
(iii) Semipre-irresolute [10] if the inverse image of each semipreopen set of Y is semipreopen in X
(iv) Gp-continuous [2] if the inverse image of each closed set of Y is gp-closed in X.
(v) gp- irresolute [2] if the inverse image of each gp-closed set of Y is gp-closed in X.
(vi) Pre-gp-continuous [16]if the inverse image of each preclosed set of Y is gp-closed in X.
(vii) gsp-continuous [4] if the inverse image of each closed set of Y is gsp-closed in X.
(viii) gsp-irresolute [4] if the inverse image of each gsp-closed set of Y is gsp-closed in X.
(ix) Pre-gsp-continuous [12] if the inverse image of each semipreopen set of Y is gsp-open in Y.
(x) αg- irresolute [3] if the inverse image of each αg-closed set of Y is αg-closed in X.
(xi) rps-continuous [19] if the inverse image of each closed set of Y is rps-closed in X.
(xii) rps-irresolute [19] if the inverse image of each rps-closed set of Y is rps-closed in X.

3. Properties of gsp-Hausdroff spaces

We, define the following.

Definition 3.1: A space X is called gsp-Hausdorff if for any pair of distinct points x,y∈X ,there exist disjoint gsp-open sets U and V such that x∈U and y∈V.

Definition 3.2: A space X is called semipre-Hausdorff if for any pair of distinct points x,y∈ X ,there exist disjoint semipreopen sets U and V such that x∈U and y∈V.
Clearly, every semipre-Hausdorff space is an gsp-Hausdorff.
We have the following invariant properties

Theorem 3.3: If f:X→Y is injective gsp-continuous and Y is Hausdorff space, then X is gsp-Housdroff.

Proof: Since f is injective , f(x) ≠f(y) for x , y ∈ X and x≠y. Now , as Y being Hausdorff space there exist open sets G and H in Y such that f(x) ∈ G ,f(y) ∈ H and G ∩ H = ϕ. Let U = f^{-1}(G) and V= f^{-1}(H). Then U and V are gsp-open sets in X ,since f is gsp-continuous function . Also ,x ∈ f^{-1}(G) = U , y ∈ f^{-1}(H) = V and U ∩ V = f^{-1}(G) ∩ f^{-1}(H) = ∅. Hence X is gsp-Hausdorff.

Theorem 3.4: If f:X→Y is injective, gsp-irresolute and Y is gsp-Hausdorff , then X is gsp- Hausdorff.

Proof: Since f is injective, f(x)≠f(y) for x,y∈X, and x≠y. Now Y being gsp-Hausdorff there exist gsp-open sets G,H in Y, such that f(x) ∈G, f(y) ∈H and G∩H=∅. Let U=f^{-1}[G] and V= f^{-1}[H]. Then U and V are gsp-open in X as f is gsp-irresolute. Also, x∈ f^{-1}[G]=U, y∈ f^{-1}[H]=V and U∩V= f^{-1}[G] ∩f^{-1}[H] = ∅. Hence X is gsp-Hausdorff.

Theorem 3.5: If f:X→Y is injective pre-gsp-continuous and Y is semipre-Hausdorff space, then X is gsp-Hausdorff.

Proof: Since f is injective, f(x) ≠f(y) for x , y ∈ X and x≠y. Now, as Y being semipre -Hausdorff space there exist semipreopen sets G and H in Y such that f(x) ∈ G ,f(y) ∈ H and G ∩ H = ϕ. Let U = f^{-1}(G) and V= f^{-1}(H). Then U and V are gsp-open sets in X ,since f is gsp-continuous function . Also ,x ∈ f^{-1}(G) = U , y ∈ f^{-1}(H) = V and U ∩ V = f^{-1}(G) ∩ f^{-1}(H) = ∅. Hence X is gsp-Hausdorff.

Theorem 3.6 : If f:X→Y is injective, semipre-continuous and Y is Hausdorff , then X is semipre-Hausdorff.

Proof: Since f is injective, f(x) ≠f(y) for x, y ∈ X and x≠y. Now, as Y being Hausdorff space there exist open sets G and H in Y such that f(x)∈ G,f(y)∈ H and G ∩ H = ϕ. Let, U = f^{-1}(G) and V= f^{-1}(H). Then U and V are semipre-open sets in X, since f is semipre-continuous function . Also, x∈ f^{-1}(G) = U, y ∈ f^{-1}(H) = V and U ∩ V = f^{-1}(G) ∩ f^{-1}(H)= ∅. Hence X is semipre-Hausdorff.

Theorem 3.7: If f:X→Y is injective, semipre-irresolute and Y is semipre-Hausdorff , then X is semipre-Hausdorff.
Proof is similar to Th.3.4.

Theorem 3.8: If f:X→Y is injective, strongly semipre-continuous and Y is semipre-Hausdorff , then X is Hausdorff.
Proof is similar to Th.3.4.

We, define the following.

Definition 3.9: A space X is called gp-Hausdorff if for each pair of distinct points x , y ∈ X ,there exist disjoint gp-open sets U and V such that x∈U and y∈V.

Definition 3.10: A space X is called p-Hausdorff if for each pair of distinct points x, y ∈ X , there exist disjoint p-open sets U and V such that x∈U and y∈V.
Clearly, every gp-Hausdorff space is gsp-Hausdorff since every gp-open set is gsp-open set.
Every p –Hausdorff space is gp-Hausdorff since every p-open set is gp-open set.
Now, we prove the following.

Theorem 3.11 : If f:X→Y is injective gp-continuous and Y is Hausdorff space, then X is gp-Hausdorff .

Proof: Since f is injective , f(x) ≠f(y) for x,y∈X and x≠y. now Y being Hausdorff space there exist open sets G and H in Y such that f(x) ∈G ,f(y) ∈H and G∩H=ϕ. Let U= f^{-1}(G) and V= f^{-1}(H). Then U and V are gp-open in X as f being gp-continuous function. Also , x∈ f^{-1}(G) =U y∈ f^{-1}(H)=V and U∩V= f^{-1}(G)∩f^{-1}(H) =ϕ. Hence X is gp-Hausdorff.

Theorem 3.12 If f:X→Y is injective gp-irresolute and Y is gp-Hausdorff space, then X is gp-Hausdorff.

Proof: Similar to Th.3.3 above.

Theorem 3.13: If f:X→Y is injective pre-irresolute and Y is p-Hausdorff space, then X is p-Hausdorff.

Proof: Similar to Th.3.3 above.

Theorem 3.14: If f:X→Y is injective pre-gp-continuous and Y is p-Hausdorff space, then X is gp-Hausdorff.

Proof: Similar to Th.3.3 above.

We, define the following

Definition 3.15: A space X is called rps-Hausdorff if for any pair of distinct points x,y∈ X ,there exist disjoint rps-open sets U and V such that x∈U and y∈V.

Clearly, (i) every semipre-Hausdorff space is an rps-Hausdorff, since every semipreopen set is rps-open set.
(ii) every rps-Hausdorff space is an gsp-Hausdorff , since every rps-open set is gsp-open set.

Definition 3.16: A function f : X→ Y is called (rps,gsp)-continuous if the inverse image of each rps-open set of Y is gsp-open in X.

Definition 3.17: A function f : X→ Y is called (gsp ,rps)-continuous if the inverse image of each gsp-open set of Y is rps-open in X.
We, state the following.

Theorem 3.18: If f:X→Y is injective (rps, gsp)-continuous and Y is rps-Hausdorff space, then X is gsp-Hausdorff.

Theorem 3.19: If f:X→Y is injective (gsp,rps)-continuous and Y is gsp-Hausdorff space, then X is rps-Hausdorff .
We define the following.

Definition 3.20: The space X is called αg-Hausdorff if and only if for x,y∈X such that x≠y there exist disjoint αg-open sets U and V such that x∈U and y∈V
Clearly, every αg-Hausdorff space ⇒ gp-Hausdorff space ⇒ gsp-Hausdorff space, since as we have , αg-closed set → gp-closed set → gsp-closed set.

Theorem 3.21: If f: X→Y is injective αg-irresolute and Y is αg-Hausdorff space, then X is αg -Housdroff.

Proof: Since f is injective , f(x) ≠f(y) for x,y∈X and x≠y. Now Y being αg-Hausdorff space there exist αg-open sets G H in Y such that f(x) ∈G,f(y) ∈H and G∩H=ϕ. Let U= f^{-1}(G) and V= f^{-1}(H). Then U and V are αg -open in X. Also x∈ f^{-1}(G) =U y∈ f^{-1}(H)=V and U∩V= f^{-1}(G)∩f^{-1}(H) =ϕ. Hence X is αg-Hausdorff.

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